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Homotopy perturbation method for motion of a spherical solid particle in plane couette fluid flow

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ABSTRACT

He's homotopy perturbation method is applied to obtain exact analytical solutions for the motion of a spherical particle in a plane couette flow. It is demonstrated that the applied analytical method is very straightforward in comparison with existing techniques. Furthermore, it is decidedly effectual in terms of accuracy and rapid convergence. The formulation of the problem is presented in the text as well as the analytical and numerical procedures. The current results can be used in different areas of particulate flows.

1. Introduction

Understanding particulate transport processes is of crucial interest to the further development of many industrial applications such as coal energy systems. Investigation of a single solid or liquid object, moving in a continuous phase, helps to clarify the behavior of these systems. Several works could be found in the technical literature, such as [1,2], which study the behavior of dispersed particles in continuous medias.

The majority of pervious works in the field of particulate and aerosol flows are performed numerically using the Eulerian–Lagrangian approach. When using a Eulerian–Lagrangian model to deal with two-phase flows, continuity and momentum balance equations governs the hydrodynamics of the continuous phase, but each discrete particle is taken as a point mass, and its velocity and position are governed by Newton's second law, $m_i \frac{du_i}{dt} = F_i$, where F_i is the resultant force exerted on a particle. The overall external force, F_i physically consists of gravity, buoyancy, other body forces and interphase interacting forces, such as drag, virtual mass force, the Basset force, lift force, *etc*. The force balance equation is solved numerically in different related papers with different methods such as the finite difference method. However, an analytical expression is more convenient for engineering calculations, and is also the obvious starting point for a better understanding of the relationship between the physical properties of the sphere-fluid combination and the accelerated motion of the sphere.

This paper aims to apply an analytical technique for the equations that govern the 2D motion of a spherical particle in plane couette fluid flow. Thus, we utilize the Homotopy Perturbation Method, HPM, as a powerful series-based analytical tool. This technique was introduced and employed for different applications in fluid mechanics by He [3,4]. The HPM was previously used to describe the motion of a single particle falling in a continuous Newtonian media [5,6].

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Vander Werff [7] proposed a constructive mathematical formulation for motion of a spherical particle in a plane couette flow. The author assumed a 2D velocity profile incompressible Newtonian flow. Considering the rotation of the particle, it was assumed that the particle will rotate with a constant angular velocity Ω given by one-half the curl of the fluid motion.

Generally, the particle's motion is determined by the combined effects of inertia, drag and lift. Gravity and buoyancy effects will be assumed negligible [7]. Subsequently, the governing equations are obtained as:

$$\frac{4}{3}\pi r^3 \rho_s \ddot{x} = \frac{1}{2}\pi r^3 \rho_f \alpha \dot{y} - 6\pi\mu r(\dot{x} - \alpha y) \tag{1}$$

and

$$\frac{4}{3}\pi r^3 \rho_s \ddot{y} = \left(\frac{1}{2}\pi r^3 \rho_f \alpha + 6.46 r^2 \rho_f \sqrt{\alpha}\sqrt{\upsilon}\right)(\alpha y - \dot{x}) - 6\pi\mu r\dot{y} \tag{2}$$

where r and ρ_s signify the particle radius and particle density, respectively and μ is the fluid viscosity. Moreover, the dots represent differentiation with respect to time. The relative velocities of the particle and fluid are considered small enough for the Stokes law of drag.

For this system to possess a nontrivial solution, nonzero initial conditions must be specified. The following might represent either injection of the particle into the fluid or statistical fluctuations: Alternatively, nonzero values of $\dot{x}$ or y could have been chosen where no generality is lost by specifying the particular (and physically more meaningful) conditions above. Eqs. (1) and (2) can be rewritten in the following forms:

$$\ddot{x} - \mathbf{A}\dot{y} + \mathbf{B}(\dot{x} - \alpha y) = 0 \tag{3}$$

$$\ddot{y} + \mathbf{B}\dot{y} + (\mathbf{A} + \mathbf{C})(\dot{x} - \alpha y) = 0 \tag{4}$$

where coefficients $\mathbf{A}$ to $\mathbf{C}$ are defined as:

$$\mathbf{A} = (3\alpha/8)(\rho_f/\rho_s) \tag{5a}$$

$$\mathbf{B} = (9\upsilon/2r^2)(\rho_f/\rho_s) \tag{5b}$$

$$\mathbf{C} = 1.542\left(\sqrt{\upsilon}\sqrt{\alpha}/r\right)\left(\rho_f/\rho_s\right). \tag{5c}$$

Eqs. (3) and (4), defining the motion of the particle in plane couette fluid flow, are linear and could be solved with different mathematical methods. As mentioned in the text, the nonzero initial conditions of equations of motion could be different for unlike situations. The following might represent either injection of the particle into the fluid or statistical fluctuations:

$$x = 0, \quad \dot{x} = u_0 \quad \text{at } t = 0 \tag{6a}$$

$$y = 0, \quad \dot{y} = \upsilon_0 \quad \text{at } t = 0. \tag{6b}$$

We employed the HPM to obtain an exact analytical solution for the considered system including, Eqs. (3)–(6).

2. Results and discussion

To simplify the solution, constants dependent on physical properties of solid–fluid combination are considered to be $\mathbf{A} = \mathbf{B} = \mathbf{C} = 1$, and $\upsilon_0 = u_0 = 1$. It should be mentioned that constants can be simply calculated dependent on physical conditions. For this circumstance, the homotopy equations can be constructed as,

$$H(x,p) = (1-p)\cdot\left(\frac{\mathrm{d}^2x(t)}{\mathrm{d}t} + \frac{\mathrm{d}x(t)}{\mathrm{d}t}\right) + p\cdot\left(\frac{\mathrm{d}^2x(t)}{\mathrm{d}t} + \frac{\mathrm{d}x(t)}{\mathrm{d}t} - \frac{\mathrm{d}y(t)}{\mathrm{d}t} - y(t)\right) \tag{7a}$$

$$H(y,p) = (1-p)\cdot\left(\frac{\mathrm{d}^2y(t)}{\mathrm{d}t} + \frac{\mathrm{d}y(t)}{\mathrm{d}t}\right) + p\cdot\left(\frac{\mathrm{d}^2y(t)}{\mathrm{d}t} + \frac{\mathrm{d}y(t)}{\mathrm{d}t} + \frac{\mathrm{d}x(t)}{\mathrm{d}t} - y(t)\right). \tag{7b}$$

Thus, p-terms with their relative solutions can be found as,

$$\begin{aligned} &p_x^0 : \frac{\mathrm{d}^2x_0(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}x_0(t)}{\mathrm{d}t} \\ &x_0(t) = 1 - e^{-t} \end{aligned} \tag{8a}$$

$$\begin{aligned} &p_y^0 : \frac{\mathrm{d}^2y_0(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y_0(t)}{\mathrm{d}t} \\ &y_0(t) = 1 - e^{-t} \end{aligned} \tag{8b}$$

$$\begin{aligned} &p_x^1 : \frac{\mathrm{d}^2x_1(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} - 1 \\ &x_1(t) = e^{-t} + t - 1 \end{aligned} \tag{9a}$$

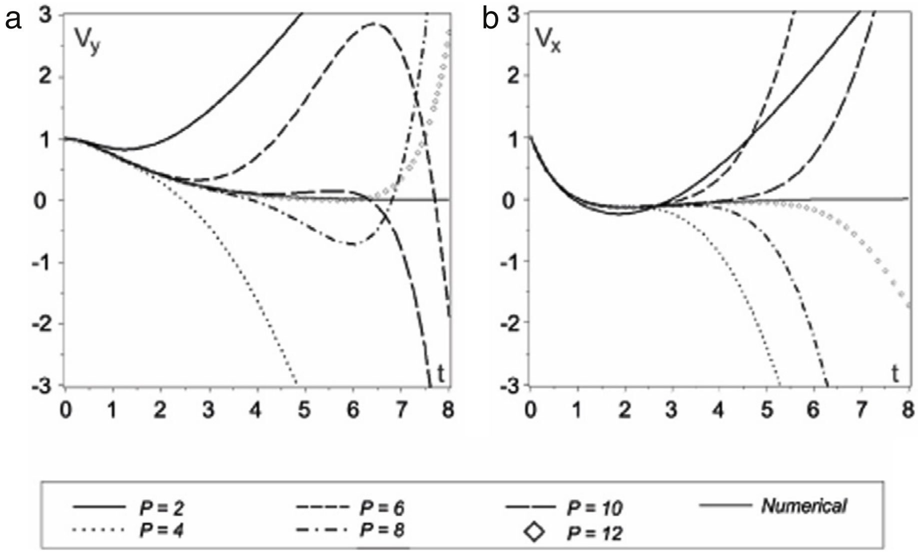

Fig. 1. Velocity variations for the HPM (different series terms) and accurate numerical solutions.

$$
\begin{aligned}
&p_y^1 : \frac{\mathrm{d}^2 y_1(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y_1(t)}{\mathrm{d}t} - 1 + 2e^{-t} \\
&y_1(t) = 2e^{-t}t + 3e^{-t} + t - 3
\end{aligned}
\tag{9b}
$$

$$
\begin{aligned}
&p_s^2 : \frac{\mathrm{d}^2 x_2(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} - 2e^{-t} - t + 2 \\
&x_2(t) = -2e^{-t}t - 5e^{-t} - 3t + \frac{1}{2}t^2 + 5
\end{aligned}
\tag{10a}
$$

$$
\begin{aligned}
&p_y^2 : \frac{\mathrm{d}^2 y_2(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y_2(t)}{\mathrm{d}t} - 4e^{-t} + 4 - 2e^{-t}t - t \\
&y_2(t) = -e^{-t}t^2 - 6e^{-t}t - 11e^{-t} + \frac{1}{2}t^2 + 11
\end{aligned}
\tag{10b}
$$

$$
\begin{aligned}
&p_x^3 : \frac{\mathrm{d}^2 x_3(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}x_3(t)}{\mathrm{d}t} + 2e^{-t}t + 6e^{-t} + 4t - \frac{1}{2}t^2 - 6 \\
&x_3(t) = \frac{1}{6}t^3 - \frac{5}{2}t^2 + e^{-t}t^2 + 8e^{-t}t + 19e^{-t} + 11t - 19
\end{aligned}
\tag{11a}
$$

$$
\begin{aligned}
&p_y^3 : \frac{\mathrm{d}^2 y_3(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y_3(t)}{\mathrm{d}t} + 6t + 8e^{-t}t + 14e^{-t} - 14 + e^{-t}t - \frac{1}{2}t^2 \\
&y_3(t) = -\frac{7}{2}t^2 + 21t + 5e^{-t}t^2 + 24e^{-t}t + 45e^{-t} + \frac{1}{3}e^{-t}t^3 + \frac{1}{6}t^3 - 45
\end{aligned}
\tag{11b}
$$

$$
\begin{aligned}
&p_x^4 : \frac{\mathrm{d}^2 x_4(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}x_4(t)}{\mathrm{d}t} + 3t^2 - 14t - e^{-t}t^2 - 10e^{-t}t - 24e^{-t} - \frac{1}{6}t^3 + 24 \\
&x_4(t) = -6e^{-t}t^2 - 36e^{-t}t - 81e^{-t} - \frac{7}{6}t^3 + \frac{21}{2}t^2 - 45t - \frac{1}{3}e^{-t}t^3 + \frac{1}{24}t^4 + 81
\end{aligned}
\tag{12a}
$$

$$
\begin{aligned}
&p_y^4 : \frac{\mathrm{d}^2 y_4(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y_4(t)}{\mathrm{d}t} + 4t^2 - 26t - 6e^{-t}t^2 - 30e^{-t}t - 56e^{-t} - \frac{1}{3}e^{-t}t^3 - \frac{1}{6}t^3 + 56 \\
&y_4(t) = -\frac{3}{2}t^3 + \frac{35}{2}t^2 - 91t - \frac{7}{3}e^{-t}t^3 - 22e^{-t}t - 100e^{-t}t - 191e^{-t} - \frac{1}{12}e^{-t}t^4 + \frac{1}{24}t^4 + 191
\end{aligned}
\tag{12b}
$$

$$\vdots$$

Fig. 1 depict the variations of horizontal and vertical velocities for the described situations.

Clearly by adding to series terms the moving particle is covered for longer.

Basically, by estimating the constants **A** to **C** for each selected combinations of solid–fluid, results can be derived easily. The results prove the accuracy and capability of the HPM for the solution of these types of problems applicable in different areas of fluid mechanics.

3. Conclusion

In the present paper, we have employed the HPM (Homotopy Perturbation Method) to solve a couple equations govern the motion of a particle in plane couette fluid flow. The results achieved by HPM are compared with those obtained by a numerical method (Runge–Kutta) and an interesting agreement is demonstrated. It was shown that by adding to the number of *p*-terms, a higher accuracy will be achieved by the HPM. So, for a specific condition, the HPM can be applied simply and a number of terms can be obtained by comparison with numerical results. Obtained results can be used in further surveys in the field of particulate flows.

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